Precalculus Online, Test 2 Review Answers

Dr. Graham-Squire

1. Bob invests \$10,000 in a bank account at 4% interest, compounded continuously. How many years will it take until he has \$21,000 in the account? Round to the nearest 0.1 years.

   Ans: 18.5 years

2. Solve the equations. Round your answer to the nearest 0.01.

   (a) $7^{x/2} = 5^{1-x}$ **Ans**: $x = 0.62$

   (b) $\log_{10} x + \log_{10}(x-3) = 1$

   Ans: $x = 5$ is the only solution. $x = -2$ does not work because it gives the log of a negative number when you plug it back in, which is not allowed.

3. The bat population in a certain region was 350,000 in 2009, and the observed doubling time for the population is 25 years. When will the population reach 2 million?

   Ans: The year 2072

4. The half-life of palladium is 4 days. After 20 days a sample has been reduced to a mass of 0.375 grams. After how many days was exactly one gram left?

   Ans: 14.33 days

5. (a) Find the reference number for $t = \frac{-35\pi}{4}$. **Ans**: $\bar{t} = \frac{\pi}{4}$

   (b) Find the terminal point for $t = \frac{41\pi}{6}$. **Ans**: $\left(\frac{-\sqrt{3}}{2}, \frac{1}{2}\right)$

6. (a) Find $\cos t$ and $\csc t$ if $\tan t = \frac{1}{4}$ and t lies in Quadrant III.

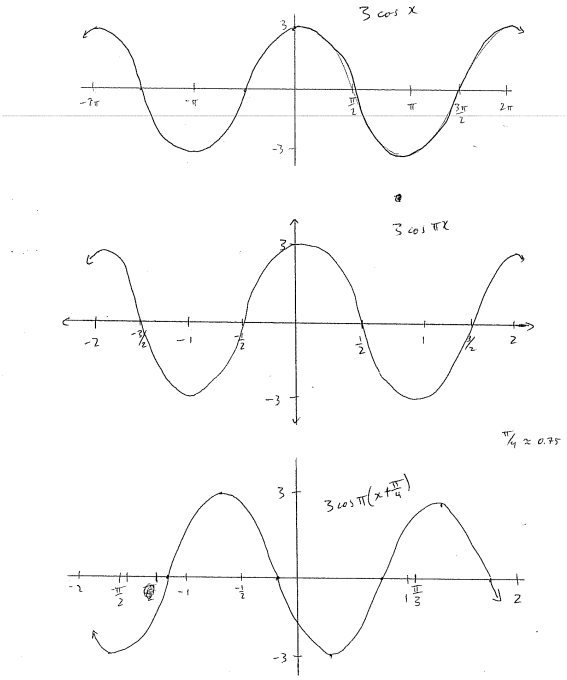
   Ans: $\cos t = -4/\sqrt{17}$, $\csc t = -\sqrt{17}$

   (b) Without a calculator, find the following. If an expression does not exist, write DNE and explain why it does not exist.

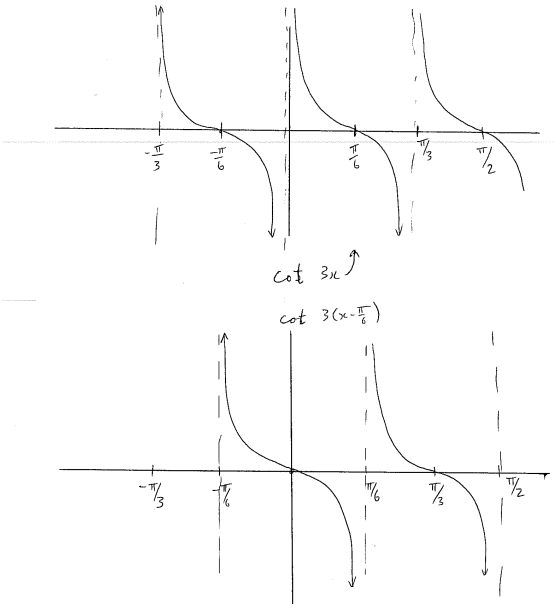
   (i) $\sin \frac{3\pi}{4} = \frac{\sqrt{2}}{2}$ (ii) $\tan \frac{-7\pi}{3} = -\sqrt{3}$

   (iii) $\sec \frac{7\pi}{2}$ = dne, because you cannot divide by zero, and cosine of $3\pi/2$ is zero.

7. Without a calculator, sketch a graph of $y = 3\cos\left(\pi\left(x + \frac{\pi}{4}\right)\right)$.



8. Without a calculator, sketch a graph of $y = \cot 3\left(x - \frac{\pi}{6}\right)$.



9. Without a calculator, find the exact value of the following. If an expression does not exist, write DNE and explain why it does not exist.

   (a) $\sin^{-1}(-\frac{\sqrt{2}}{2}) = \frac{-\pi}{4}$

   (b) $\cos^{-1}(1) = 0$

   (c) $\tan^{-1}(1) = \frac{\pi}{4}$

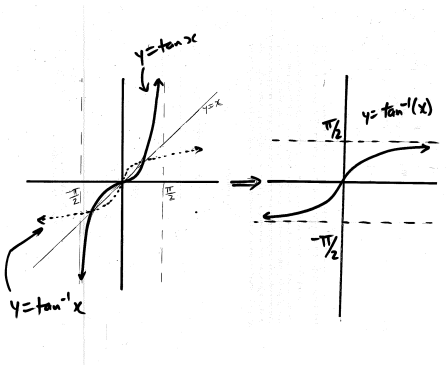
   (d) $\cos^{-1}(\frac{\pi}{2}) =$ DNE because $\frac{\pi}{2} > 1$

   (e) $\tan^{-1}(-\sqrt{3}) = \frac{-\pi}{3}$

   (f) $\tan^{-1}(\tan\left(\frac{4\pi}{3}\right)) = \frac{\pi}{3}$

   (g) $\tan(\sin^{-1}(1)) = \tan(\frac{\pi}{2}) =$ DNE because tangent of pi/2 is not defined (you would be dividing by zero because $\cos\frac{\pi}{2} = 0$.

10. Sketch a graph of the tangent function for x-values between $-\pi/2$ and $\pi/2$. Now draw the line $y = x$, and flip the graph of tangent over the line $y = x$ to sketch the graph of $y = \tan^{-1} x$



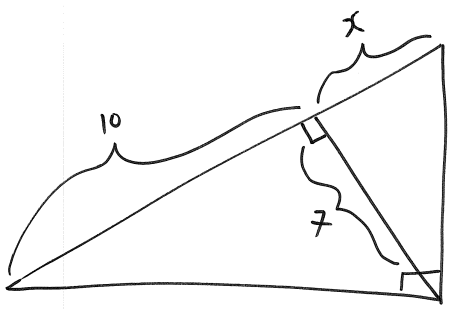
11. New York and Los Angeles are 2450 miles apart. Thinking of that distance as an arc lying on a circle, find the angle that the arc subtends at the center of the earth. (Note: you will need to use the fact that the radius of the earth is 3960 miles). Find the angle in both radians and degrees, round to the nearest 0.1.

    Ans: 0.618 radians and 35.4 degrees

12. Solve the right triangle with a hypotenuse of length 20 and one angle equal to 53°. Round to the nearest 0.1.

    Ans: the two sides are 16 and 12, and the angle is 37 degrees.

13. Solve for x. Round to the nearest 0.1.



Ans: You have to first use the sides 7 and 10 to set up an equation such as $\tan\theta = \frac{7}{10}$, then use the inverse tangent to find the value of the angle θ. Use that angle to find the other angle in the big triangle, then use tangent to solve for $x = 4.9$.

14. A plane is flying at an elevation of 5000 feet, directly above a straight highway. Two cars are on the highway on opposite sides of the plane. The angle of depression to one car is 35° and the angle of depression to the other car is 48°. How far apart are the cars? Round to the nearest 0.1 feet.

    Ans: 11642.7

15. Write $\sec\theta$ in terms of $\sin\theta$, assuming θ is in Quadrant 2.

    Ans: $\dfrac{1}{-\sqrt{1-\sin^2\theta}}$

16. Find the exact value of $\sin(\tan^{-1}\dfrac{11}{8})$, without using a calculator. Then write it as a decimal number and compare to what you get on a calculator.

    Ans: $11/\sqrt{185}$, or 0.8087 in decimal form.

17. Beatrice is playing "make a triangle", one of her favorite games. She walks 36 feet from point M to point N, then turns 57° and walks in a straight line to point P. She then turns 94° and walks in a straight line back to point M. At which point she sits down and starts crying, because she realizes that she forgot to measure the distances for lines $\overline{NP}$ and $\overline{MP}$, as well as angle M. Console Beatrice by calculating all of those values for her without rewalking the triangle.

    Ans: Use the Law of Sines to calculate $\overline{NP} = 17.5$ feet, $\overline{MP}$=30.3 feet, and angle $M = 29°$.

18. Quadrilateral $ABCD$ has the following dimensions: $\overline{AD} = 13$, $\overline{AB} = 11$, $\overline{BC} = 17$, $\overline{CD} = 8$, and angle A is 38°. Let $\overline{BD}$ be the diagonal that cuts $ABCD$ into two triangles. Calculate the measurement of angle CDB *or* explain why such a quadrilateral does not exist. Back up any conclusions you make with mathematical calculations and/or explanations.

    Ans: Such a quadrilateral does NOT exist! Looking at the triangle ADB, you can use the law of cosines to find that the diagonal BD must have length 8.04. But this means that the triangle DBC must have sides 8, 8.04, and 17, which is impossible because 8+8.04 does not add up to be more than 17! Alternatively, if you try to do the law of cosines on triangle DBC to find the angle CDB, you will end up trying to solve $\cos C = -1.24$, which is impossible because cosine only has values between one and negative one.